

Exhibit K

FEEDBACK CONTROL OF *Dynamic Systems*

FOURTH EDITION



GENE F. FRANKLIN
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Feedback Control of Dynamic Systems

Fourth Edition

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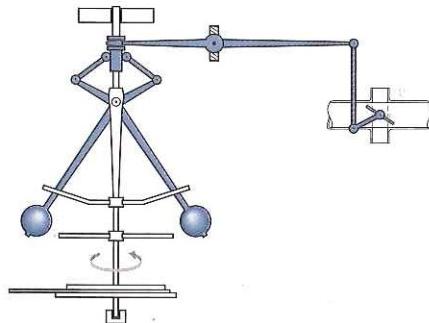
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1 An Overview and Brief History of Feedback Control



Chapter Overview

In this chapter we begin our exploration of feedback control using a simple familiar example: a household furnace controlled by a thermostat. The generic components of a control system are identified within the context of this example. In another example—an automobile cruise control—we develop the elementary static equations and assign numerical values to elements of the system model in order to compare the performance of open-loop control to that of feedback control when dynamics are ignored. In order to provide a context for our studies and to give you a glimpse of how the field has evolved, Section 1.3 provides a brief history of control theory and design. Finally, Section 1.4 provides a brief overview of the contents and organization of the entire book.

A Perspective on Feedback Control

Open-loop control

Feedback control

Control of dynamic systems is a very common concept with many characteristics. A system that involves a person controlling a machine, as in driving an automobile, is called **manual control**. A system that involves machines only, as when room temperature can be set by a thermostat, is called **automatic control**. Systems designed to hold an output steady against unknown disturbances are called **regulators**, while systems designed to track a reference signal are called **tracking** or **servo** systems. Control systems are also classified according to the information used to compute the controlling action. If the controller does *not* use a measure of the system output being controlled in computing the control action to take, the system is called **open-loop control**. If the controlled output signal *is* measured and fed back for use in the control computation, the system is called closed-loop or **feedback control**. There are many other important properties of control systems in addition to these most basic characteristics. For example, in this book we will mainly be concerned with controlling processes that can be adequately described by **linear, time invariant** equations, whereas all physical processes are nonlinear if the signals are large and their characteristics vary with time if observed for a long time. We will also mainly consider feedback of the present output only, but a very familiar example illustrates the limitation imposed by that assumption. When driving a car, the use of simple feedback corresponds to driving in a thick fog where one can *only see the road immediately at the front of the car* and is unable to see the future required position! Looking at the road ahead is a form of predictive control. This information, which has obvious advantages, would always be used where it is available; but in most automatic control situations studied in this book, observation of the future track or disturbance is not possible. In any case the control designer should study the process to see if any sensor could anticipate either a track to be followed or a disturbance to be rejected. If such a possibility is feasible, the control designer should use it to **feed forward** an early warning to the control system. An example of this is in the control of steam pressure in the boiler of an electric power generation plant. A measure of the *electric* power demand at the output of the plant can be fed forward to the boiler controller in anticipation of a soon-to-be-demanded increase in steam flow.

2 Chapter 1 An Overview and Brief History of Feedback Control

The evolution of cheap and powerful digital computers has had a major impact on control design and control implementation. Software such as MATLAB is a great aid to solving the equations and realizing the graphics of control design methods. For analyzing system response, students of control have tools such as Simulink which can easily compute the response of linear as well as nonlinear models of processes and controls. The controllers which compute the signals necessary to effect control are mainly electronic units because of the flexibility and cost-effectiveness of these devices. While analog units are typically faster and cheaper to implement simple equations than digital logic, the greater programming flexibility and increasing cost-effectiveness of embedded digital processors is causing them to become ever more common in controller implementation. The influence of these trends on our introduction to the stimulating field of control is evident throughout the text.

The applications of feedback control have never been more exciting than they are today. Landing and collision avoidance systems using the Global Positioning System (GPS) are now under development and promise a revolution in our ability to navigate in an ever more crowded airspace. In the magnetic data storage devices for computers known as hard disks, control of the read/write head assembly is often designed to have tracking errors on the order of microns and to move at speeds of a fraction of a millisecond. Control is essential to the operation of systems from cell phones to jumbo jets and from washing machines to oil refineries as large as a small city. The list goes on and on. In fact, many engineers refer to control as a *hidden technology* because of its essential importance to so many devices and systems while being mainly out of sight. The future will no doubt see engineers create even more imaginative applications of feedback control. Study of control problems over the past 200 years has led to an extensive body of knowledge common to both manual and automatic control which has evolved into the discipline of control systems design, the subject of this book.

1.1 A Simple Feedback System

In feedback systems the variable being controlled—such as temperature or speed—is measured by a sensor, and the measured information is fed back to the controller to influence the controlled variable. The principle is readily illustrated by a very common system, the household furnace controlled by a thermostat. The components of this system and their interconnections are shown in Fig. 1.1(a). Such a picture identifies the major parts of the system and shows the directions of information flow from one component to another.

We can easily analyze the operation of this system qualitatively from the graph. Suppose both the temperature in the room where the thermostat is located and the outside temperature are significantly below the reference temperature (also called the set point) when power is applied. The thermostat will be *on*, and the control logic will open the furnace gas valve and light the fire

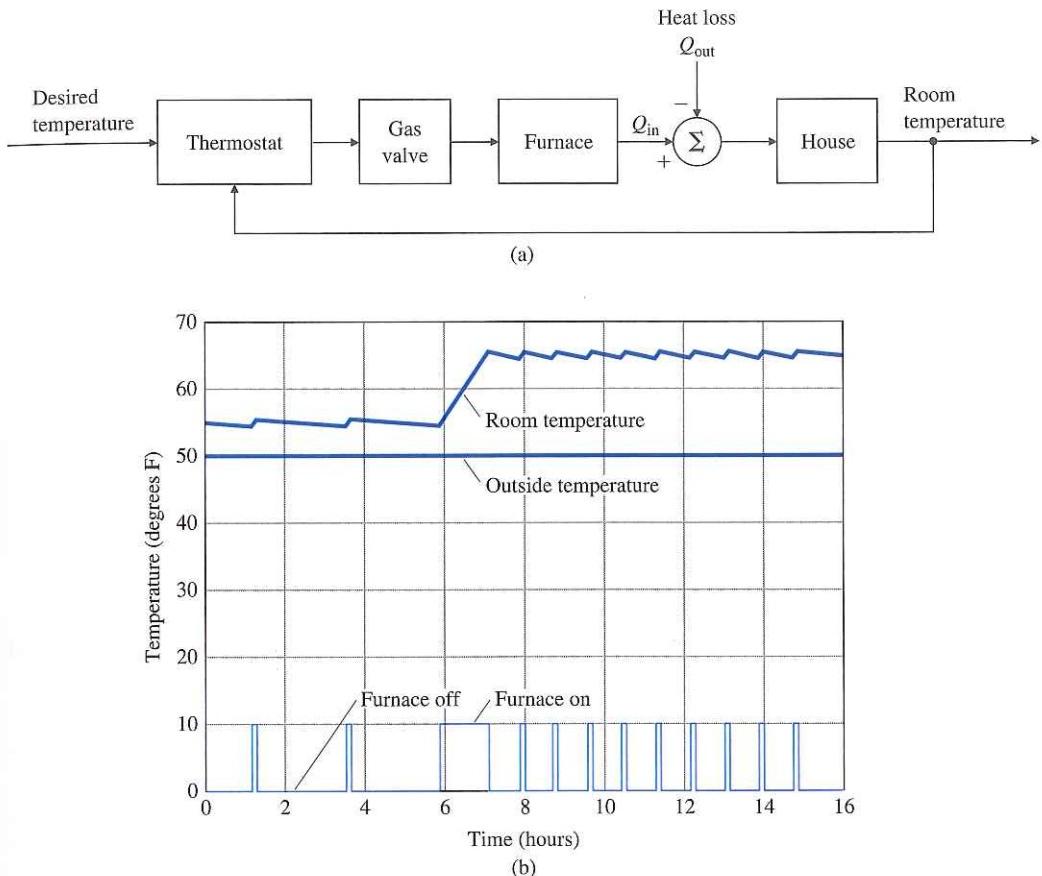


Figure 1.1 (a) Component block diagram of a room temperature control system (b) Plot of room temperature and furnace action

box. This will cause heat Q_{in} to be supplied to the house at a rate that will be significantly larger than the heat loss Q_{out} . As a result, the room temperature will rise until it exceeds the thermostat reference setting by a small amount. At this time the furnace will be turned off and the room temperature will start to fall toward the outside value. When it falls a small amount below the set point, the thermostat will come on again and the cycle will repeat. Typical plots of room temperature along with the furnace cycles of on and off are shown in Fig. 1.1. The outside temperature is held at 50°F and the thermostat is initially set at 55°F. At 6 a.m., the thermostat is stepped to 65°F and the furnace brings it to that level and cycles the temperature around that figure thereafter.¹ Notice that the house is well-insulated so that the fall of temperature with the furnace

¹ Notice that the furnace had come on a few minutes before 6 a.m. on its regular nighttime schedule.

4 Chapter 1 An Overview and Brief History of Feedback Control

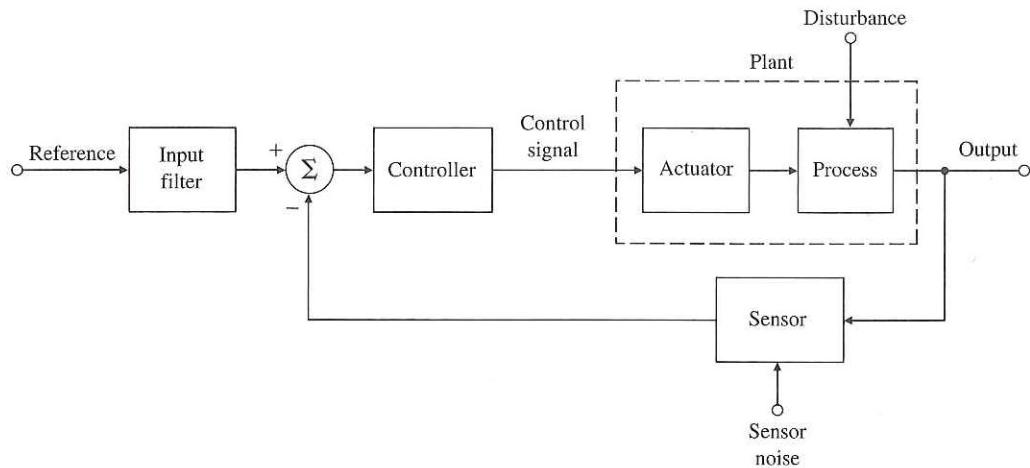


Figure 1.2 Component block diagram of an elementary feedback control

off is significantly slower than the rise with the furnace on. From this example we can identify the generic components of the elementary feedback control system as shown in Fig. 1.2.

The central component of this feedback system is the **process** whose output is to be controlled. In our example the process would be the house whose output is the room temperature, and the **disturbance** to the process is the flow of heat from the house due to conduction through the walls and roof to the lower outside temperature. (The outward flow of heat also depends on other factors such as wind, open doors, etc.) The design of the process can obviously have a major impact on the effectiveness of the controls. The temperature of a well-insulated house with thermopane windows is clearly easier to control than otherwise. Similarly, the design of aircraft with control in mind makes a world of difference to the final performance. In every case, the earlier the issues of control are introduced into the process design, the better. The **actuator** is the device that can influence the controlled variable of the process, and in our case the actuator is a gas furnace. Actually, the furnace usually has a pilot light or striking mechanism, a gas valve, and a blower fan which turns on or off depending on the air temperature in the furnace. These details illustrate the fact that many feedback systems contain components that themselves form other feedback systems.² The central issue with the actuator is its ability to move the process output with adequate speed and range. The furnace must produce more heat than the house loses on the worst day and must distribute it quickly if the house temperature is to be kept in a narrow range. Power, speed, and reliability are usually more important than accuracy. Generally, the process

² Jonathan Swift (1733) said it this way: "So, Naturalists observe, a flea Hath smaller fleas that on him prey; And these have smaller still to bite 'em; And so proceed, *ad infinitum*."

Section 1.1 A Simple Feedback System 5

and the actuator are intimately connected and the control design centers on finding a suitable input or control signal to send to the actuator. The combination of process and actuator is called the **plant**, and the component that actually computes the desired control signal is the **controller**. Because of the flexibility of electrical signal processing, the controller typically works on electrical signals, although the use of pneumatic controllers based on compressed air has a long and important place in process control. With the development of digital technology, cost effectiveness and flexibility have led to the use of digital signal processors as the controller in an increasing number of cases. The component labeled **thermostat** in Fig. 1.1 measures the room temperature and is called the **sensor** in Fig. 1.2, a device whose output inevitably contains sensor noise. Sensor selection and placement are very important in control design because it is sometimes not possible for the true controlled variable and the sensed variable to be the same. For example, although we may really wish to control the house temperature as a whole, the thermostat is in one particular room, which may or may not be at the same temperature as the rest of the house. For instance, if the thermostat is set to 68°F but is placed in the living room near a roaring fireplace, a person working in the study could still feel uncomfortably cold.^{3,4}

As we will see in addition to placement, important properties of a sensor are the accuracy of the measurements as well as low noise, reliability, and linearity. The sensor will typically convert the physical variable into an electrical signal for use by the controller. Our general system also includes an **input shaping filter** whose role is to convert the reference signal to electrical form for later manipulation by the controller. In some cases the input shaping filter can modify or shape the reference command input in ways that improve the system response. Finally, there is a **comparator** to compute the difference between the reference signal and the sensor output to give the controller a measure of the system error.

This text will present methods for analyzing feedback control systems and their components and will describe the most important design techniques engineers can use with confidence in applying feedback to solve control problems. We will also study the specific advantages of feedback that compensate for the additional complexity it demands. However, although the temperature control system is easy to understand, it is nonlinear as seen by the fact that the furnace is either on or off and to introduce linear controls we need another example.

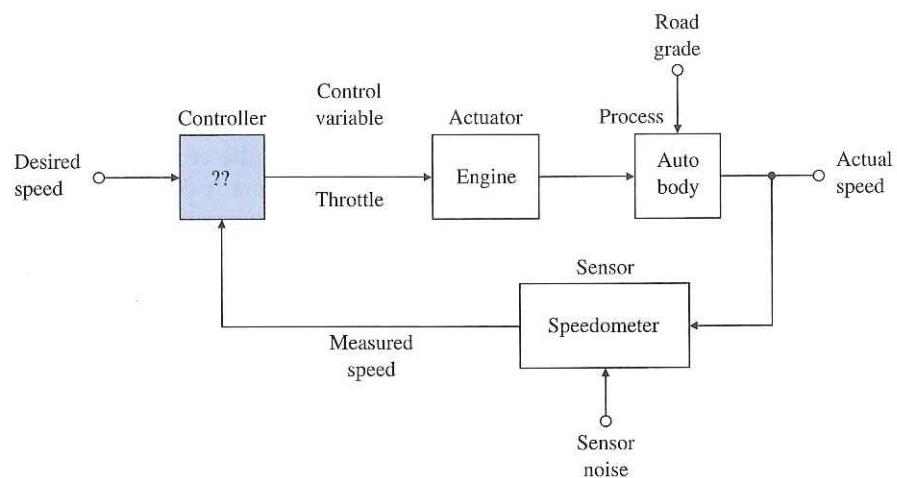
³ In the renovations of the kitchen in the house of one of the authors the new ovens were placed against the wall where the thermostat was mounted on the other side. Now when dinner is baked in the kitchen on a cold day, the author freezes in his study unless the thermostat is reset.

⁴ The story is told of the new employee at the nitroglycerin factory who was to control the temperature of a critical part of the process manually. He was told to “keep that reading below 300°.” On a routine inspection tour, the supervisor realized that the batch was dangerously hot and found the worker holding the thermometer under the cold water tap to bring it down to 300°. They got out just before the explosion. Moral: Sometimes automatic control is better than manual.

6 Chapter 1 An Overview and Brief History of Feedback Control

Figure 1.3

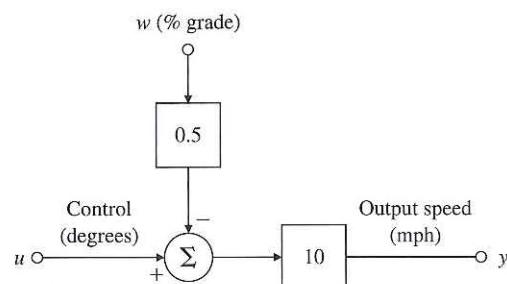
Component block diagram
of automobile cruise control

**1.2 A First Analysis of Feedback**

The value of feedback can be readily demonstrated by quantitative analysis of a simplified model of a familiar system, the cruise control of an automobile (Fig. 1.3). To study this situation analytically, we need a mathematical **model** of our system in the form of a set of quantitative relationships among the variables. For this example we ignore the dynamic response of the car and consider only the steady behavior. (Dynamics will of course play a major role in later chapters.) Furthermore, we assume that for the range of speeds to be used by the system, we can approximate the relations as linear. After measuring the speed of the vehicle on a level road at 65 mph, we find that a 1° change in the throttle angle (our control variable) causes a 10-mph change in speed. From observations while driving up and down hills it is found that when the grade changes by 1%, we measure a speed change of 5 mph. The speedometer is found to be accurate to a fraction of 1 mph and will be considered exact. With these relations, we can draw the **block diagram** of the plant (Fig. 1.4), which shows these mathematical relationships in graphical form. In this diagram the connecting lines carry signals and a block is like an ideal amplifier which

Figure 1.4

Block diagram of the cruise control plant



Section 1.2 A First Analysis of Feedback 7

Open-loop control

multiplies the signal at its input by the value marked in the block to give the output signal. To sum two or more signals, we show lines for the signals coming into a summer, a circle with the summation sign Σ inside. An algebraic sign (plus or minus) beside each arrow head indicates whether the input adds to or subtracts from the total output of the summer. For this analysis, we wish to compare the effects of a 1% grade on the output speed when the reference speed is set for 65 with and without feedback to the controller.

In the first case shown in Fig. 1.5 the controller does not use the speedometer reading but sets $u = r/10$. This is an example of an **open-loop control system**. The term *open-loop* refers to the fact that there is no closed path or loop around which the signals go in the block diagram. In our simple example the open-loop output speed, y_{ol} , is given by the equations

$$\begin{aligned}y_{ol} &= 10(u - 0.5w) \\&= 10\left(\frac{r}{10} - 0.5w\right) \\&= r - 5w.\end{aligned}$$

The error in output speed is

$$e_{ol} = r - y_{ol} \quad (1.1)$$

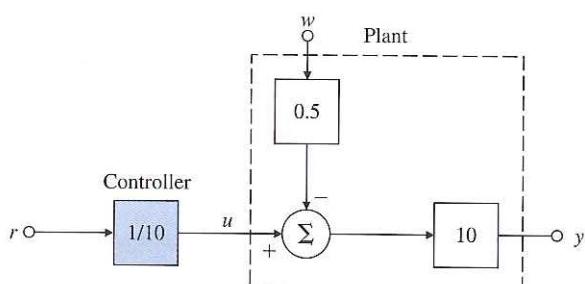
$$= 5w, \quad (1.2)$$

and the percent error is

$$\% \text{ error} = 500 \frac{w}{r}. \quad (1.3)$$

If $r = 65$ and the road is level, then $w = 0$ and the speed will be 65 with no error. However, if $w = 1$ corresponding to a 1% grade, then the speed will be 60 and we have a 5-mph error, which is a 7.69% error in the speed. For a grade of 2%, the speed error would be 10 mph, which is an error of 15.38%, and so on. The example shows that there would be no error when $w = 0$, but this result depends on the controller gain being the exact inverse of the plant gain of 10. In practice, the plant gain is subject to change; and if it does change, errors are introduced by this means also. If there is an error in the plant gain in open-loop control, the percent speed error would be the same as the percent plant-gain error.

Figure 1.5
Open-loop cruise control



8 Chapter 1 An Overview and Brief History of Feedback Control

The block diagram of a feedback scheme is shown in Fig. 1.6, where the controller gain has been set to 10. Recall that in this simple example, we have assumed that we have an ideal sensor whose block is not shown. In this case the equations are

$$y_{cl} = 10u - 5w,$$

$$u = 10(r - y_{cl}).$$

Combining them yields

$$y_{cl} = 100r - 100y_{cl} - 5w,$$

$$101y_{cl} = 100r - 5w,$$

$$y_{cl} = \frac{100}{101}r - \frac{5}{101}w,$$

$$e_{cl} = \frac{r}{101} + \frac{5w}{101}.$$

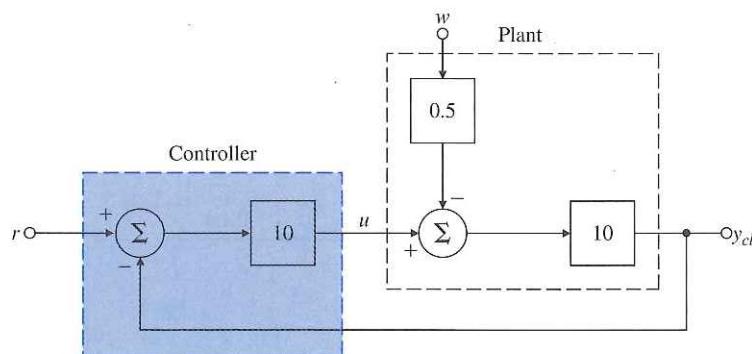
Thus the feedback has reduced the sensitivity of the speed error to the grade by a factor of 101 when compared with the open-loop system. Note, however, that there is now a small speed error on level ground, because even when $w = 0$,

$$y_{cl} = \frac{100}{101}r = 0.99r \text{ mph.}$$

This error will be small as long as the loop gain (product of plant and controller gains) is large.⁵ If we again consider a reference speed of 65 mph and compare

Figure 1.6

Closed-loop cruise control



⁵ In case the error is too large, it is common practice to *reset* the reference—in this case to $\frac{101}{100}r$ —so the output reaches the true desired value.

speeds with a 1% grade, the percent error in the output speed is

$$\%error = 100 \frac{\frac{65 \times 100}{101} - \left(\frac{65 \times 100}{101} - \frac{5}{101} \right)}{\frac{65 \times 100}{101}} \quad (1.4)$$

$$= 100 \frac{5 \times 101}{101 \times 65 \times 100} \quad (1.5)$$

$$= 0.0769\%. \quad (1.6)$$

The design tradeoff

The reduction of the speed sensitivity to grade disturbances and plant gain in our example is due to the loop gain of 100 in the feedback case. Unfortunately, there are limits to how high this gain can be made; when dynamics are introduced, the feedback can make the response worse than before, or even cause the system to become unstable. The dilemma is illustrated by another familiar situation where it is easy to change a feedback gain. If one tries to raise the gain of a public-address amplifier too much, the sound system will squeal in a most unpleasant way. This is a situation where the gain in the feedback loop—from the speakers to the microphone through the amplifier back to the speakers—is too much. The issue of how to get the gain as large as possible to reduce the errors without making the system become unstable and squeal is what much of feedback control design is all about.

Liquid-level control

1.3 A Brief History

An interesting history of early work on feedback control has been written by O. Mayr (1970), who traces the control of mechanisms to antiquity. Two of the earliest examples are the control of flow rate to regulate a water clock and the control of liquid level in a wine vessel, which is thereby kept full regardless of how many cups are dipped from it. The control of fluid flow rate is reduced to the control of fluid level, since a small orifice will produce constant flow if the pressure is constant, which is the case if the level of the liquid above the orifice is constant. The mechanism of the liquid-level control invented in antiquity and still used today (for example, in the water tank of the ordinary flush toilet) is the **float valve**. As the liquid level falls, so does the float, allowing the flow into the tank to increase; as the level rises, the flow is reduced and, if necessary, cut off. Figure 1.7 shows how a float valve operates. Notice here that sensor and actuator are not separate devices but are, instead, contained in the carefully shaped float-and-supply-tube combination.

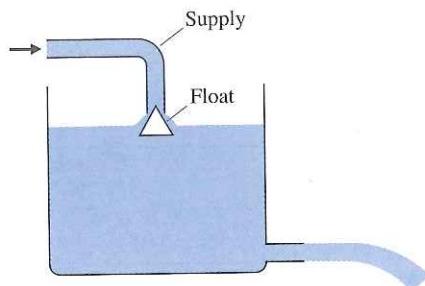
Drebble's incubator

A more recent invention described by Mayr (1970) is a system, designed by Cornelis Drebble in about 1620, to control the temperature of a furnace

10 Chapter 1 An Overview and Brief History of Feedback Control

Figure 1.7

Early historical control of liquid level and flow

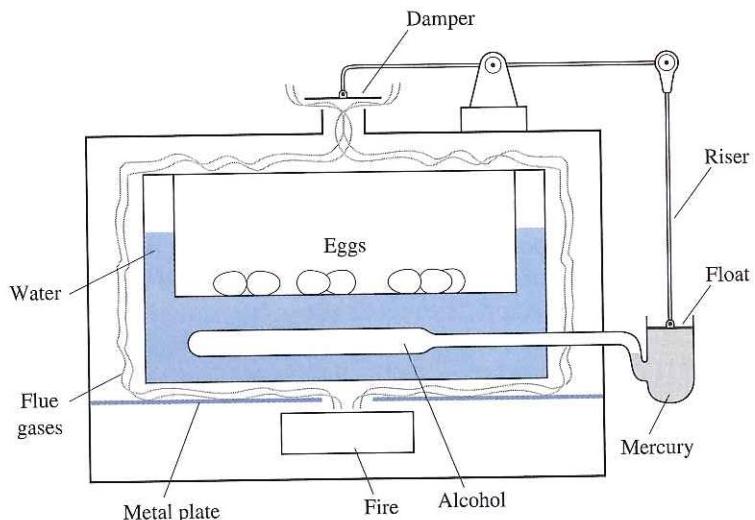


used to heat an incubator⁶ (Fig. 1.8). The furnace consists of a box to contain the fire, with a flue at the top fitted with a damper. Inside the fire box is the double-walled incubator box, the hollow walls of which are filled with water to transfer the heat evenly to the incubator. The temperature sensor is a glass vessel filled with alcohol and mercury and placed in the water jacket around the incubator box. As the fire heats the box and water, the alcohol expands and the riser floats up, lowering the damper on the flue. If the box is too cold, the alcohol contracts, the damper is opened, and the fire burns hotter. The desired temperature is set by the length of the riser, which sets the opening of the damper for a given expansion of the alcohol.

A famous problem in the chronicles of control systems was the search for a means to control the rotation speed of a shaft. Much early work (Fuller, 1976) seems to have been motivated by the desire to automatically control the speed of the grinding stone in a wind-driven flour mill. Of various meth-

Figure 1.8

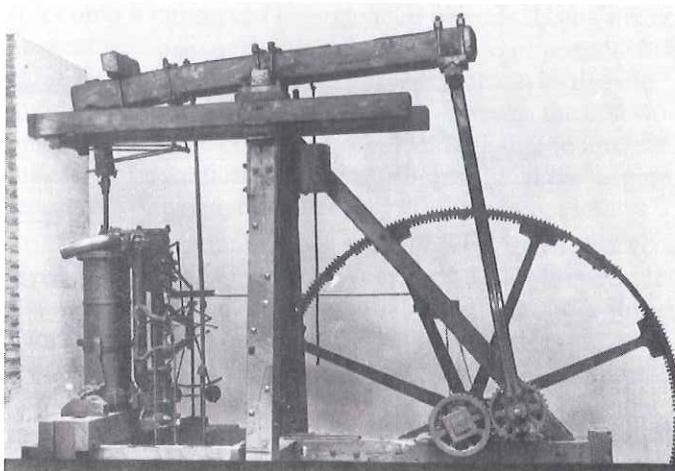
Drebbel's incubator for hatching chicken eggs.
(Adapted from Mayr, 1970)



⁶ French doctors introduced incubators into the care of premature babies over 100 years ago.

Figure 1.9

A steam engine from the shop of James Watt.
(*British Crown Copyright, Science Museum, London*)

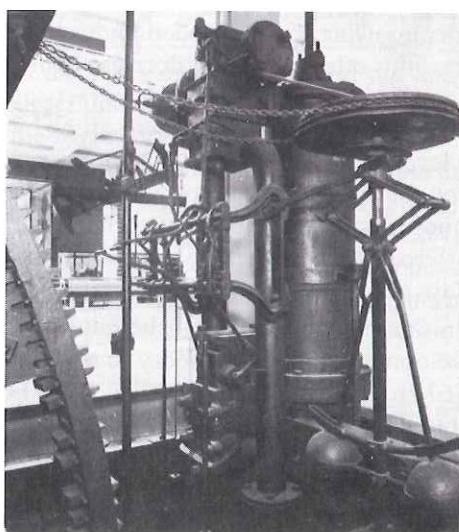
**Fly-ball governor**

ods attempted, the one with the most promise used a conical pendulum, or **fly-ball governor**, to measure the speed of the mill. The sails of the driving windmill were rolled up or let out with ropes and pulleys, much like a window shade, to maintain fixed speed. However, it was adaptation of these principles to the steam engine in the laboratories of James Watt around 1788 that made the fly-ball governor famous. An early version is shown in Fig. 1.9, while Figs. 1.10 and 1.11 show a close-up of a fly-ball governor and a sketch of its components.

The action of the fly-ball governor (also called a centrifugal governor) is simple to describe. Suppose the engine is operating in equilibrium. Two

Figure 1.10

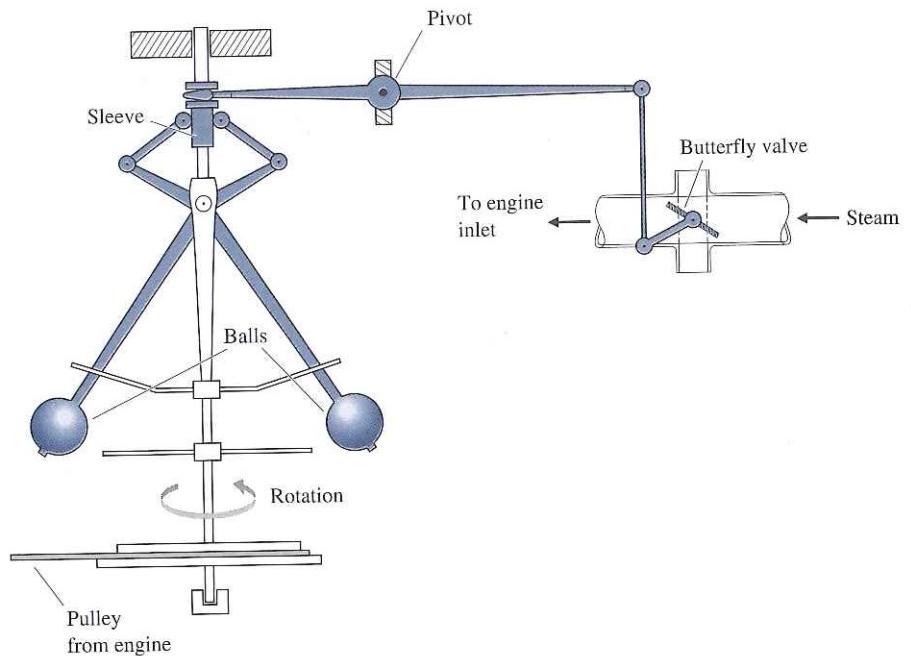
Watt's steam engine
(1789–1800) with fly-ball
governor. (*British Crown
Copyright, Science
Museum, London*)



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Figure 1.11

Operating parts of a fly-ball governor



weighted balls spinning around a central shaft can be seen to describe a cone of a given angle with the shaft. When a load is suddenly applied to the engine, its speed will slow, and the balls of the governor will drop to a smaller cone. Thus the ball angle is used to sense the output speed. This action, through the levers, will open the main valve to the steam chest (which is the actuator) and admit more steam to the engine, restoring most of the lost speed. To hold the steam valve at a new position it is necessary for the fly balls to rotate at a different angle, implying that the speed under load is not exactly the same as before. We saw this effect earlier with cruise control, where feedback control gave a very small error. To recover the exact same speed in the system, it would require resetting the desired speed setting by changing the length of the rod from the lever to the valve. Subsequent inventors introduced mechanisms that integrated the speed error to provide automatic reset. In Chapter 4 we will analyze these systems to show that such integration can result in feedback systems with zero steady-state error to constant disturbances.

Because Watt was a practical man, like the millwrights before him, he did not engage in theoretical analysis of the governor. Fuller (1976) has traced the early development of control theory to a period of studies from Christian Huygens in 1673 to James Clerk Maxwell in 1868. Fuller gives particular credit to the contributions of G. B. Airy, professor of mathematics and astronomy at Cambridge University from 1826 to 1835 and Astronomer Royal at Greenwich Observatory from 1835 to 1881. Airy was concerned with speed control; if his telescopes could be rotated counter to the rotation of the Earth, a fixed

star could be observed for extended periods. Using the centrifugal-pendulum governor he discovered that it was capable of unstable motion: “and the machine (if I may so express myself) became perfectly wild” (Airy, 1840; quoted in Fuller, 1976). According to Fuller, Airy was the first worker to discuss instability in a feedback control system and the first to analyze such a system using differential equations. These attributes signal the beginnings of the study of feedback control dynamics.

The first systematic study of the stability of feedback control was apparently given in the paper “On Governors” by J. C. Maxwell (1868).⁷ In this paper, Maxwell developed the differential equations of the governor, linearized them about equilibrium, and stated that stability depends on the roots of a certain (characteristic) equation having negative real parts. Maxwell attempted to derive conditions on the coefficients of a polynomial that would hold if all the roots had negative real parts. He was successful only for second- and third-order cases. Determining criteria for stability was the problem for the Adams Prize of 1877, which was won by E. J. Routh.⁸ His criterion, developed in his essay, remains of sufficient interest that control engineers are still learning how to apply his simple technique. Analysis of the characteristic equation remained the foundation of control theory until the invention of the electronic feedback amplifier by H. S. Black in 1927 at Bell Telephone Laboratories.

Shortly after publication of Routh’s work, the Russian mathematician A. M. Lyapunov (1893) began studying the question of stability of motion. His studies were based on the nonlinear differential equations of motion and also included results for linear equations that are equivalent to Routh’s criterion. His work was fundamental to what is now called the state-variable approach to control theory but was not introduced into the control literature until about 1958.

The development of the feedback amplifier is briefly described in an interesting article based on a talk by H. W. Bode (1960) reproduced in Bellman and Kalaba (1964). With the introduction of electronic amplifiers, long-distance telephoning became possible in the decades following World War I. However, as distances increased, so did the loss of electrical energy; in spite of using larger diameter wire, increasing numbers of amplifiers were needed to replace the lost energy. Unfortunately, large numbers of amplifiers resulted in much distortion because the small nonlinearity of the vacuum tubes then used in electronic amplifiers were multiplied many times. To solve the problem of reducing distortion, Black proposed the feedback amplifier. As mentioned earlier in connection with the automobile cruise control, the more we wish to reduce errors (or distortion), the more feedback we need to apply. The loop gain from actuator to plant to sensor to actuator must be made very large.

⁷ An exposition of Maxwell’s contribution is given in Fuller (1976).

⁸ E. J. Routh was first academically in his class at Cambridge University in 1854, while J. C. Maxwell was second. In 1877 Maxwell was on the Adams Prize Committee that chose the problem of stability as the topic for the year.

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With high gain the feedback loop begins to squeal and is unstable. Here was Maxwell's and Routh's stability problem again, except that in this technology the dynamics were so complex (with differential equations of order 50 being common) that Routh's criterion was not very helpful. So the communications engineers at Bell Telephone Laboratories, familiar with the concept of frequency response and the mathematics of complex variables, turned to complex analysis. In 1932 H. Nyquist published a paper describing how to determine stability from a graphical plot of the loop frequency response. From this theory there developed an extensive methodology of feedback-amplifier design described by Bode (1945) and extensively used still in the design of feedback controls. Nyquist and Bode plots are discussed in more detail in Chapter 6.

Simultaneous with the development of the feedback amplifier, feedback control of industrial processes was becoming standard. This field, characterized by processes that are not only highly complex but also nonlinear and subject to relatively long time delays between actuator and sensor, developed **proportional integral-derivative (PID) control**. The PID controller was first described by Callender et al. (1936). This technology was based on extensive experimental work and simple linearized approximations to the system dynamics. It led to standard experiments suitable to application in the field and eventually to satisfactory "tuning" of the coefficients of the PID controller. (PID controllers are covered in Chapter 4.) Also under development at this time were devices for guiding and controlling aircraft; especially important was the development of sensors for measuring aircraft altitude and speed. An interesting account of this branch of control theory is given in McRuer (1973).

An enormous impulse was given to the field of feedback control during World War II. In the United States, engineers and mathematicians at the MIT Radiation Laboratory combined their knowledge to bring together not only Bode's feedback amplifier theory and the PID control of processes but also the theory of stochastic processes developed by N. Wiener (1930). The result was the development of a comprehensive set of techniques for the design of **servomechanisms**, as control mechanisms came to be called. Much of this work was collected and published in the records of the Radiation Laboratory by James et al. (1947).

Another approach to control systems design was introduced in 1948 by W. R. Evans, who was working in the field of guidance and control of aircraft. Many of his problems involved unstable or neutrally stable dynamics, which made the frequency methods difficult, so he suggested returning to the study of the characteristic equation that had been the basis of the work of Maxwell and Routh nearly 70 years earlier. However, Evans developed techniques and rules allowing one to follow graphically the paths of the roots of the characteristic equation as a parameter was changed. His method, the **root locus**, is suitable for design as well as for stability analysis and remains an important technique today. The root-locus method developed by Evans is covered in Chapter 5.

During the 1950s several authors, including R. Bellman and R. E. Kalman in the United States and L. S. Pontryagin in the U.S.S.R., began again to consider the ordinary differential equation (ODE) as a model for control systems. Much

PID control

Root locus

State-variable design

of this work was stimulated by the new field of control of artificial earth satellites, in which the ODE is a natural form for writing the model. Supporting this endeavor were digital computers, which could be used to carry out calculations unthinkable 10 years before. (Now, of course, these calculations can be done by any engineering student with a desktop computer.) The work of Lyapunov was translated into the language of control at about this time, and the study of optimal controls, begun by Wiener and Phillips during World War II, was extended to optimizing trajectories of nonlinear systems based on the calculus of variations. Much of this work was presented at the first conference of the newly formed International Federation of Automatic Control held in Moscow in 1960.⁹ This work did not use the frequency response or the characteristic equation but worked directly with the ODE in “normal” or “state” form and typically called for extensive use of computers. Even though the foundations of the study of ODEs were laid in the late 19th century, this approach is now often called **modern control** to distinguish it from **classical control**, which uses the complex variable methods of Bode and others. In the period from the 1970s continuing through the present, we find a growing body of work that seeks to use the best features of each technique.

Thus we come to the current state of affairs where the principles of control are applied in a wide range of disciplines, including every branch of engineering. The well-prepared control engineer needs to understand the basic mathematical theory that underlies the field and must be able to select the best design technique suited to the problem at hand. With the ubiquitous use of computers it is especially important that the engineer is able to use his or her knowledge to guide and verify calculations done on the computer.¹⁰

Modern control
Classical control

1.4 An Overview of the Book

The central purpose of this book is to introduce the most important techniques for single-input-single-output control systems design. **Chapter 2** will review the techniques necessary to obtain models of the dynamic systems that we wish to control. These include model making for mechanical, electric, electromechanical, and a few other physical systems. Also described in Chapter 2 is the linearization of nonlinear models.

In **Chapter 3** and **Appendix A** we will discuss the analysis of dynamic response using Laplace transforms along with the relationship between time response and the poles and zeros of a transfer function. The chapter also

⁹ Optimal control gained a large boost when Bryson and Denham (1962) showed that the path of a supersonic aircraft should actually dive at one point in order to reach a given altitude in minimum time. This nonintuitive result was later demonstrated to skeptical fighter pilots in flight tests.

¹⁰ For more background on the history of control, see the survey papers appearing in the *IEEE Control Systems Magazine* of November 1984 and June 1996.

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includes a discussion of the critical issue of system stability including the Routh test. The chapter includes a brief discussion of numerical simulation and how to construct models from experimental data.

In **Chapter 4** we will cover the basic equations and features of feedback. An analysis of the effects of feedback on disturbance rejection, tracking accuracy, sensitivity to parameter changes, and on dynamic response will be given. The idea of elementary proportional-integral-derivative (PID) control is discussed. Also in this chapter a brief introduction is given to the digital implementation of transfer functions and thus of linear time-invariant controllers so that the effects of digital control can be compared with analog controllers as these are designed.

In **Chapters 5, 6, and 7** we introduce the techniques for realizing the control objectives first identified in Chapter 4 in more complex dynamic systems. These methods include the root locus, frequency response, and state-variable techniques. These are alternative means to the same end and have different advantages and disadvantages as guides to design of controls. The methods are fundamentally complementary, and each needs to be understood to achieve the most effective control systems design.

In **Chapter 8** we develop further the ideas of implementing controllers in a digital computer that were introduced in Chapter 4. The chapter addresses how one “digitizes” the control equations developed in Chapters 5 through 7, how the analysis of sampled systems requires another analysis tool—the z -transform—and how the sampling introduces a delay that tends to destabilize the system.

Application of all the techniques to problems of substantial complexity are discussed in **Chapter 9**. There all the design methods are brought to bear simultaneously on specific case studies.

Control designers today make extensive use of computer-aided control systems design software that is commercially available. Furthermore, most instructional programs in control systems design make software tools available to the students. The most widely used software for the purpose is MATLAB and Simulink from The Mathworks. MATLAB routines have been included throughout the text to help illustrate this method of solution and many problems require computer aids for solution. Many of the figures in the book were created using MATLAB and the files for their creation are available free of charge on the web at the sites

<http://www.prenhall.com/franklin>

or

<http://www.scsolutions.com/scsolutions.control.html>.

Students and instructors are invited to use these files because it is believed that they should be helpful in learning how to use computer methods to solve control problems.

Needless to say, many topics are not treated in the book. We do not extend the methods to multivariable controls (which are systems with more than

one input and/or output). Nor is optimal control treated in more than a very introductory manner in Chapter 7. Despite the fact that essentially all real design problems are for nonlinear plants, we have omitted any real consideration of nonlinear control, although we include brief sections in Chapters 5, 6, and 7 to illustrate the first steps in extending the several techniques to nonlinear systems. For example, the issue of the anti-windup controller is presented in Chapter 4, a brief treatment of saturation is given in Chapter 5, a discussion of the describing function is in Chapter 6, and the Lyapunov stability theory is introduced in Chapter 7.

Also beyond the scope of this text is a detailed treatment of the experimental testing and modeling of real hardware, which is the ultimate test of whether any design really works. The book concentrates on analysis and design of linear controllers for linear plant models—not because we think that is the final test of a design, but because that is the best way to grasp the basic ideas of feedback and is usually the first step in arriving at a satisfactory design. We believe that mastery of the material here will provide a foundation of understanding on which to build knowledge of these more advanced and realistic topics—a foundation strong enough to allow one to build a personal design method in the tradition of all those who worked to give us the knowledge we present here.

SUMMARY

- **Control** is the process of making a system variable adhere to a particular value, called the **reference value**. A system designed to follow a changing reference is called **tracking control** or a **servo**. A system designed to maintain an output fixed regardless of the disturbances present is called a **regulating control** or a **regulator**.
- Two kinds of control were defined and illustrated based on the information used in control and named by the resulting structure. In **open-loop control** the system does *not* measure the output and there is no correction of the actuating signal to make that output conform to the reference signal. In **closed-loop control** the system includes a sensor to measure the output and uses **feedback** of the sensed value to influence the control variable.
- A simple feedback system consists of the **process** whose output is to be controlled, the **actuator** whose output causes the process output to change, **reference** and **output sensors** that measure these signals, and the **controller**, which implements the logic by which the control signal that commands the actuator is calculated.
- **Block diagrams** are helpful for visualizing system structure and the flow of information in control systems. The most common block diagrams represent the mathematical relationships among the signals in a control system
- The theory and design techniques of control have come to be divided into two categories: **Classical control** methods use the Laplace or Fourier transforms and were the dominant methods for control design until about 1960,

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whereas **Modern control** methods are based on ordinary differential equations (ODEs) in state form and were introduced into the field starting in the 1960s. Many connections have been discovered between the two categories, and well-prepared engineers must be familiar with both techniques.

Review Questions

1. What are the main components of a feedback control system?
2. What is the purpose of the sensor?
3. Give three important properties of a good sensor.
4. What is the purpose of the actuator?
5. Give three important properties of a good actuator.
6. What is the purpose of the compensator? Give the input(s) and output(s) of the compensator.
7. What physical variable(s) of a process can be directly measured by a Hall effect sensor?
8. What physical variable is measured by a tachometer?
9. Describe three different techniques for measuring temperature.
10. Why do most sensors have an electrical output, regardless of the physical nature of the variable being measured?

Problems

- 1.1.** Draw a component block diagram for each of the following feedback control systems.
- (a) The manual steering system of an automobile
 - (b) Drebbel's incubator
 - (c) The water level controlled by a float and valve
 - (d) Watt's steam engine with fly-ball governor

In each case, indicate the location of the elements listed below and give the units associated with each signal.

- The process
- The process desired output signal
- The sensor
- The actuator
- The actuator output signal
- The controller
- The controller output signal
- The reference signal
- The error signal

Notice that in a number of cases the same physical device may perform more than one of these functions.



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